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Parametric models of tensegrity structures with double curvature

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Abstract

The current work addresses double layer tensegrity structures with double curvature, in an effort to enrich their typological configurations and to develop processes, that will facilitate their full-scale application. It constitutes part of an ongoing research, performed on a Ph.D. level. This study is based on previous research on the exploration of the geometric rules and processes that generate tensegrity structures of vaulted and spherical shape, composed of identical tensegrity units. The main goal of the current research is the development of parametric models for a double-layer tensegrity structure of spherical and ellipsoidal surface, composed of square-base units, with variable sizes. As the above-mentioned tensegrity structures, occur from the assembly of tensegrity units of square-base, the geometric approach that will be followed will permit the concurrent arrangement of the bases of the tensegrity units, on the two layers of the structure. For the developed geometric solution that permits the distribution of the square-bases on the first layer of structures of spherical and ellipsoidal shape, an application of the Mercator projection technique has been used. For the arrangement of the square-bases of the units on the second layer, various methods have been investigated and evaluated. Furthermore, the applicability of the developed methods on surfaces with more complex forms, such as minimal surfaces is addressed. The entire design process is computationally encoded and performed within the environment of Grasshopper. The main features of the developed methods and the geometric processes, are described in the paper.

Keywords

Double-layer tensegrity structures, Planar square-bases, Mercator projection, Spherical surface, Ellipsoidal surface, Minimal surfaces, Parametric models.

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Introduction

The term tensegrity structures, as defined by Fuller, refers to a special type of tensile structures, that consist of "isolated compression members (bars) and a continuous path of tension members (cables) that connect all nodes" (Fuller, 1973). Yet, even before the term was defined by Fuller, sculptural objects with the properties of tensegrity structures, were built by Kenneth Snelson (Snelson, 1965) who is probably the first one who invented this type of structure. Since the invention of the concept, significant research has been conducted to address both the geometric properties and the associated mechanical and structural properties of tensegrity structures. As far as the applications of the tensegrity concept are concerned, the geometric complexity that characterizes most tensegrity configurations, bears responsibility for delays in their adoption in the building industry.

In earlier research, Liapi has identified and demonstrated the geometric principles and rules applied to regular curved tensegrity configurations, composed of identical tensegrity units (Liapi, 2001). Based on the developed geometric rules and processes that facilitate the study of various morphologies, a parametric design method for the automatic generation of double-layer tensegrity structures in a graphical environment, using algorithmic procedures, have also been developed (Liapi and Kim, 2004) (Figure 1). The algorithm takes into account several interrelated parameters, such as the proportions and dimensions of the units that compose the structure, the overlap between the upper and lower bases of adjacent units, and the curvature of the surface. Liapi has also developed a licensed technology for the rapid on site assembly of tensegrity structures. A main feature of the method is the use of deployable units that can be easily collapsed and transported. An important advantage of the developed technology is that the collapsible and re-usable units permit the on-site assembly of structures of more than one geometric configurations (Liapi, 2005) (Figure 2).

In order to broaden the range of possible applications of this technology, this paper discusses in detail a geometric solution for the development of double-layer tensegrity structures with double curvature of less regular geometry than the aforementioned spherical and vaulted geometries. The structures are composed of square-base tensegrity units (the four vertices of each base should remain coplanar). In this study, the constraint according to which only one size of identical square-based units was required has been waived, and instead various sizes are allowed. The applicability of the method to other categories of surfaces, such as certain types of minimal surfaces (catenoid and helicoid) is also discussed. The outcome of the research is presented in the following sections.

The main goal of the research, is the development of parametric models of double curvature configurations of tensegrity structures, that can used for the construction of full-scale prototype tensegrity structure. As tensegrity structures occur from the assembly of square-base tensegrity units, a geometric approach that permits the arrangement of the bases of the units on the two layers of the structure is followed and relevant algorithms have been generated. Furthermore, the environment of Grasshopper has been used for the computational encoding process. In the following sections, the main steps of the geometric approach are described, beginning with the development of the first layer of a tensegrity structure of the spherical shape, and then of the first layer of the ellipsoid and continuing with the creation of the second layer of both structures.

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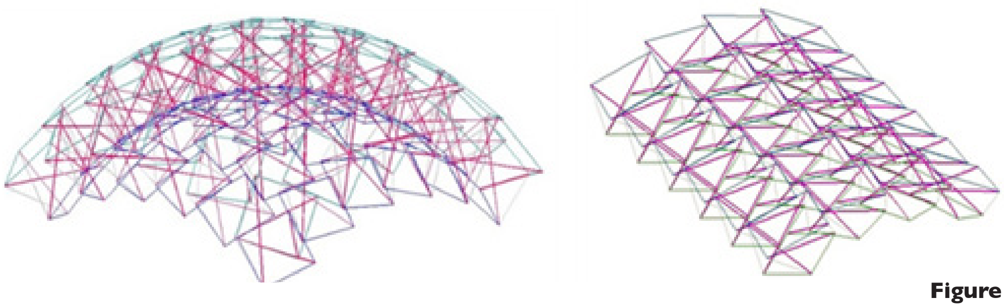

Figure 1.
Tensegrity structures composed of square-base tensegrity units: a) Spherical configurations and b) Cylindrical configurations

Design Method

The surfaces examined have been selected because of their geometric interest and their possible use in environmental applications. Specifically, a primary solar radiation analysis has been made, which indicated that when various double curvature surfaces with integrated solar panels of square shape have been compared, the ellipsoid presented better results (Figure 3). For similar reasons, minimal surfaces have been considered, because of their geometric advantages that make them particularly interesting for architectural applications.

To start the design process, the geometry of the investigated forms through physical models has been analyzed and explored (Figure 4). Digital models have also been developed and specific geometric topics that focus on various methods for the study of the distribution of the square surfaces on a layer, have been investigated, including projection methods (Liapi, Papantoniou and Nousias, 2016).

A geometric method for the development of spherical configurations of double-layer tensegrity structures composed of non-identical units, has been developed initially. In solving the first layer of the structure, among the examined methods, a technique that is based on the Mercator projection, also named isocylindrical projection, has been applied. Using this technique, the uniform arrangement of the squares of the spherical structure was made possible. For tensegrity structures of ellipsoidal shape, approximation methods were also used.

The entire design process has been computationally encoded and performed within the environment of Grasshopper. The programming languages of Python and MATLAB for the code implementation were also used. The developed method can be applied both to an oblate (flattened) ellipsoid (Figure 5b), with semi-major axis a and b, where $a=b$ (x and y-axis), and semi-minor axis c (z-axis) pointing towards the direction of compression, where $a>c$, as well as to a prolate (elongated) ellipsoid (Figure 5c), where $a<c$.

The last step in the process, that is still under development, is an interactive design tool that takes as input general geometric characteristics of the structure and returns various models of it. The steps of the developed method are described in detail in the following sections.

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Figure 2.
Deployable technology developed by Liapi: a) Deployable tensegrity unit (Full-scale model) b) Detail of a node connection and c) Structures composed of deployable units

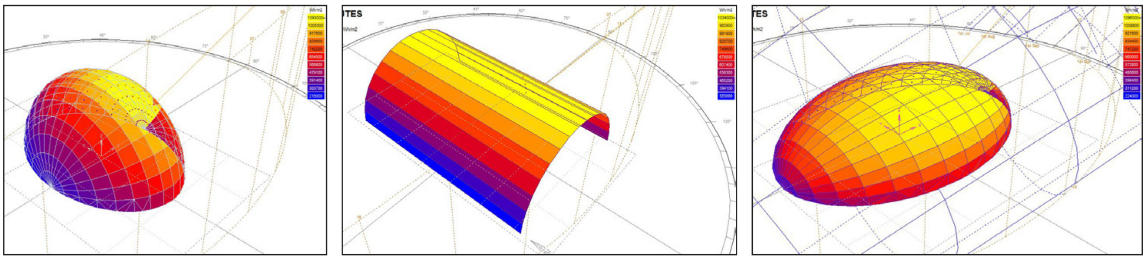

Figure 3.
Solar radiation analysis on different geometric surfaces using the program Ecotect Analysis

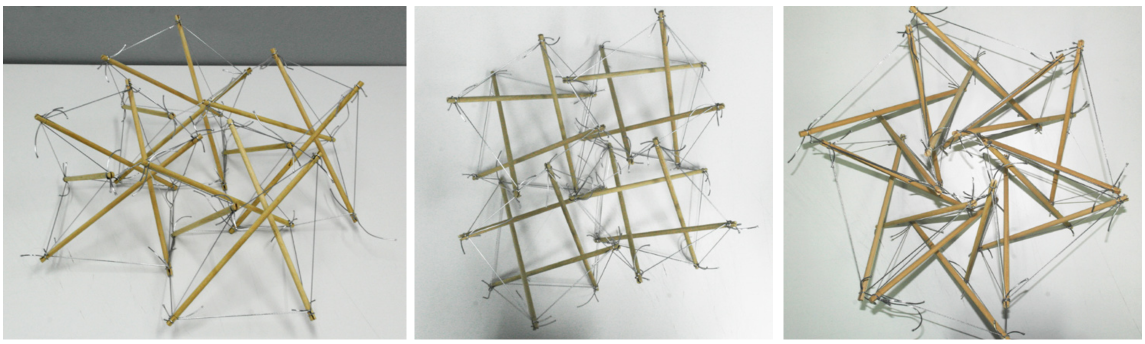

Figure 4.
Assembly of four units: mock up models forming different surfaces

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Development of the First Layer of the Structure

Spherical Tensegrity Structure

A method for the uniform distribution of the squares on a spherical surface was the first stage of the developed process. The method is based on a projection procedure from the plane to the sphere, using an analytical approach. According to the developed method, in order to simplify the problem, we replace the squares with their center points. The distances between the points need to be properly determined, ensuring that when the squares are designed around the points, their sides will be connected properly. Firstly, we attempted to project a defined orthogonal grid of equidistant points on the plane with coordinates P(x, y), where the domains for x and y are [-π, π] and [-π/2, π/2] respectively, to a sphere. After applying the spherical parametric equations $x=R\cos\phi\cos\lambda$, $y=R\cos\phi\sin\lambda$ and $z=R\sin\phi$, where ϕ and λ are the latitude and longitude of P'(λ, ϕ) respectively, a network of points is created on the sphere, and when connecting these points with lines the meridians and parallels are generated. We noticed that while the distances of the points along each parallel is constant, along the meridians the distances of the points towards the poles are increasing and as a result, the squares designed around the points became rectangles, as they came closer to the poles (Figure 6).

For the redistribution of the created points on the sphere, a technique, which is based on the Mercator projection is applied. This technique is a conformal projection; has been used in cartography for map creation and can project the parallels and meridians of the sphere to a plane (Deakin, 2002; Osborne, 2013). Practically, applying this projection we can increase the number of parallels while moving towards the poles, in order to create squares instead of rectangles. In order to proceed from the plane's coordinates to the sphere's coordinates, an inverse process of the Mercator projection has to be followed. Inverting the Mercator projection equations which are:

$$x = R\lambda\ ,\ \ y(\phi) = R\psi(\phi) = R\ln\left(\ \tan(\frac{\pi}{4}+\frac{\phi}{2})\right)$$

we can easily get the sphere coordinate λ. Also, applying an inverse equation of the Mercator projection, called Gudermannian function $gd(x)$=arctansinh$x$, we can get the coordinate ϕ. The new distribution of the points allows the proper creation of the adjacent squares (Figure 7).

In this way, we create a network of squares that are placed around these points. In order to generate the necessary 'void' between each set of four squares, each square is rotated by the same angle around a perpendicular axis that passes through its center (Figure 8). The rotation angle depends on the amount of the overlaps between the sides of the adjacent squares and the necessary scaling, in order to maintain their connections. It can be observed that, as the rotation angle increases, the overlap of the adjacent unit sides is getting lower, and respectively the void space between adjacent squares increases (Figures 9, 10).

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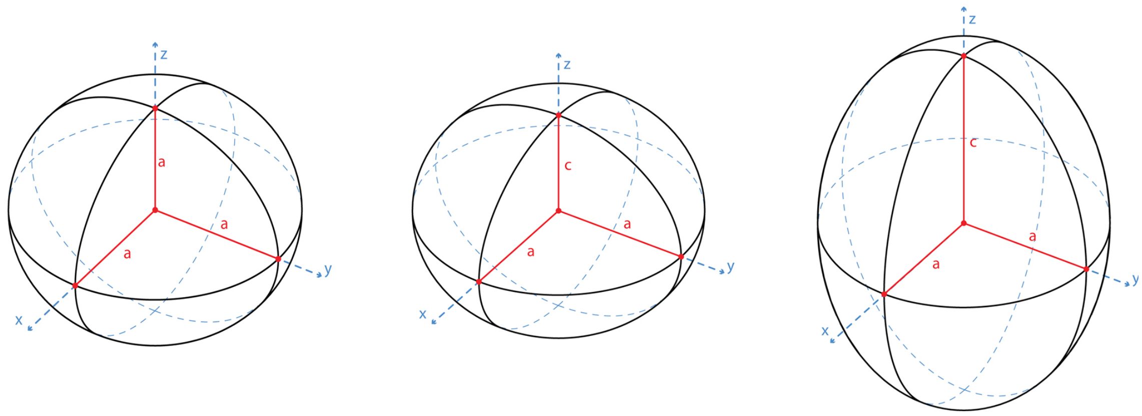

Figure 5.
Surfaces of investigation a) Sphere, b) Oblate ellipsoid and c) Prolate ellipsoid

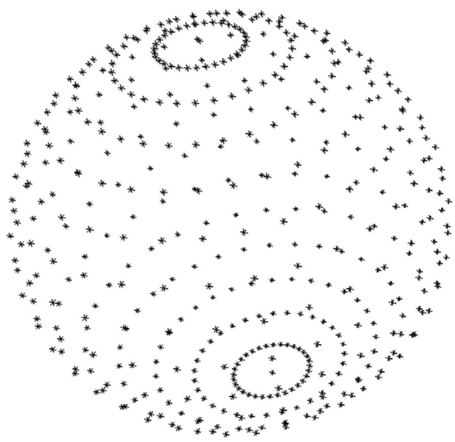

Figure 6.
Point distribution on the sphere with the use of parametric equations

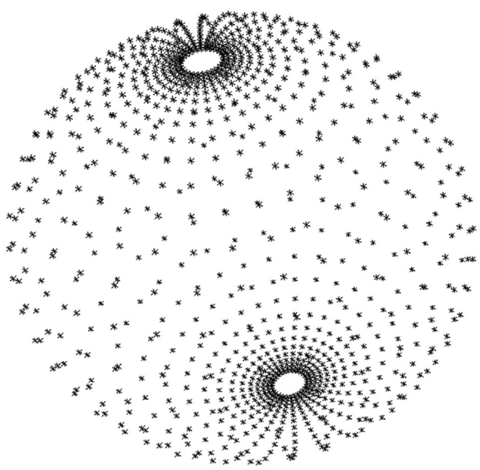

Figure 7.
Point distribution on the sphere after the application of the Mercator projection

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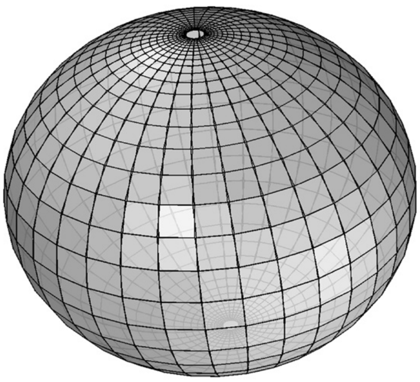

Figure 8.
Creation of the square-surfaces on the sphere

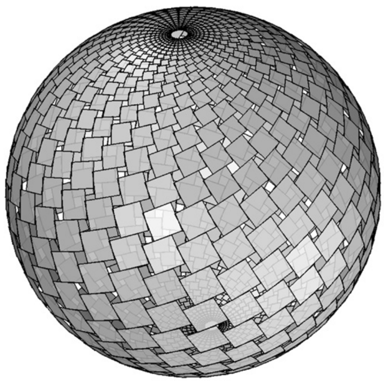

Figure 9.
Rotation of the square-surfaces

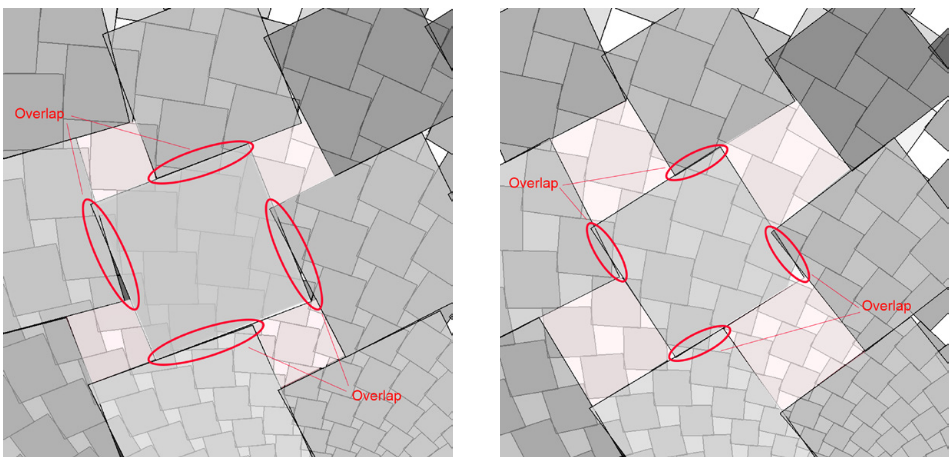

Figure 10.
Two cases of rotating the tensegrity units by different angles. The rotation angle affects the produced overlap and the void between four adjacent units

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Ellipsoidal Tensegrity Structure

The method presented in the previous section is modified for the ellipsoidal surface, by a process according to which the developed grid of points on the sphere, is projected on the ellipsoid, specifically to the ellipsoid of revolution given by the equation:

$$\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1, a>b$$

For this process, we also apply the Mercator projection technique, however further actions are required, in order to define the new point coordinates on the ellipsoid. Inverting the Mercator projection equations of the ellipsoid $x(\lambda,\phi)=\alpha\lambda$, $y(\lambda,\phi)=\alpha\psi(\phi)$, we can find the longitude λ with the same exactly way as on the sphere, however to find the latitude ϕ from $\psi(\phi)=y/\alpha$, is anything but trivial and thus, it is not possible to find a purely analytical solution.

Various techniques have been examined in order to proceed with the appropriate geometric transformations. A method that approaches the problem graphically, using numerical approximations is also developed. Specifically, in order to approximate the point position change rate along the axes, sinusoidal curves are created to represent this proportion, which are translated afterwards to polynomial expressions.

Following the projection approach and trying to find a more systematic solution, approximation methods have been developed, in order to approach the latitude ϕ, as a purely analytical solution was not possible. To address this problem, two different methods are applied, the first utilizes polynomial series and the second fixed point iterations (Osborne, 2013). Both methods give us great results throughout the entire surface, while their accuracy depends on the number of the polynomial terms and the number of iterations respectively.

After defining the values for ϕ the ellipsoidal parametric equations are applied to generate successfully the grid of points on the ellipsoid:

$$x=\rho(\phi)\cos\lambda=\nu(\phi)\cos\phi\cos\lambda,\ y=\rho(\phi)\sin\lambda=\nu(\phi)\cos\phi\sin\lambda,\ z=(1-e^2)\nu(\phi)\sin\phi$$

Consequently, the squares are created around these points, applying the same process with the sphere. It is shown that the accuracy of the method is not affected when a) the number of the squares on each row increases and b) the ratio between the two main axes of the ellipsoid changes (Figures 11, 12).

The applicability of the developed method to surfaces of less regular geometry, such as certain types of minimal surfaces has also been examined. Minimal surfaces are defined as surfaces with zero mean curvature and may also be characterized as surfaces of minimal surface area for given boundary conditions (Weisstein). Because of their particular geometric properties, the geometric solution of these surfaces can be approached easily by applying the parametric equations of each surface, and no projection technique is required. We conclude, that these types of surfaces constitute a simpler case and the developed method can be applied successfully (Figure 13).

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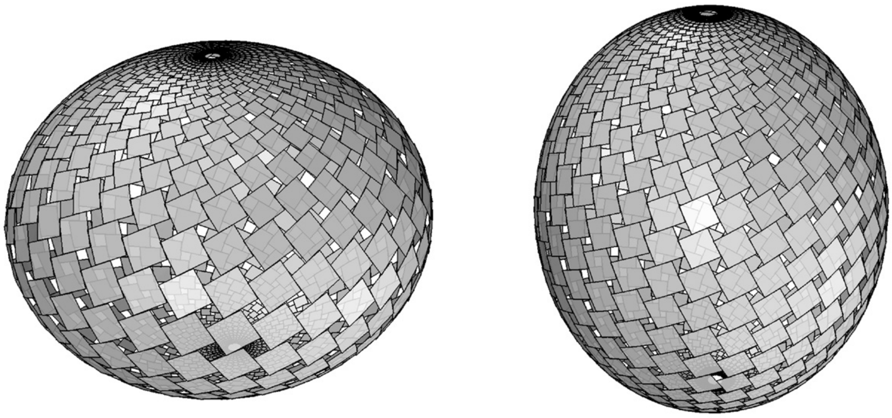

Figure 11. Arrangement of square surfaces on an ellipsoid for different values of the semi-minor axis of the ellipsoid: a) Oblate ellipsoid and b) Prolate ellipsoid

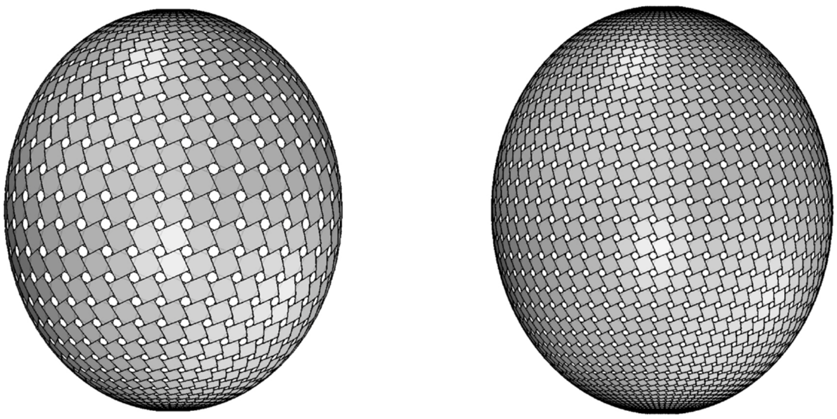

Figure 12. Arrangement of square surfaces on an ellipsoid for different numbers of squares on each row. Increasing the number of the square tiles on each row does not affect the accuracy of the method

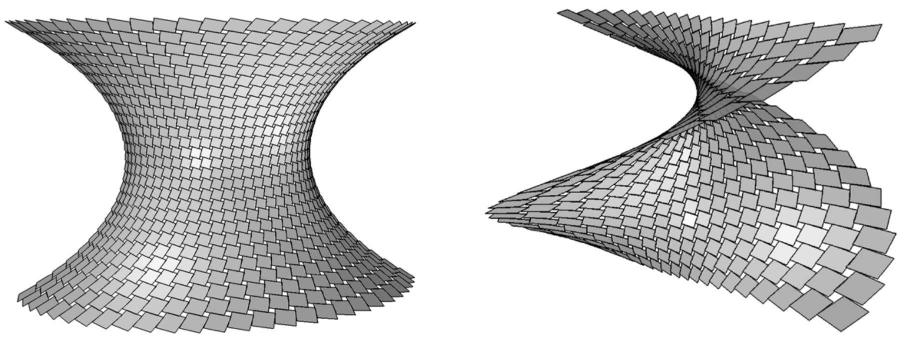

Figure 13. Creation of the first layer of a a) Catenoid and b) Helicoid

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Development of the Second Layer of the Structure

Specific requirements for the development of the structure's second layer have been determined in earlier research and have been applied in this study. In order to create the square-bases of the units on the new layer, the already formed surfaces need to be shifted on an offset surface. The centroids of the new squares need to lie on the same perpendicular axes with the first ones. As in the first layer, the vertices of each square need to be coplanar. The new squares have to be rotated by 45 degrees to the initial ones, around their perpendicular axis that passes through their centroids and make the necessary scaling concurrently (Figure 14).

Spherical Tensegrity Structure

The sphere's second layer creation is the simplest case of applying all the aforementioned requirements, without facing any difficulty. We have first created an offset surface to the initial one, the distance of which represents the thickness of the structure. The squares have been shifted on the new surface along their centroids' perpendicular axis. In the case of the sphere, these perpendicular axes passes also through the centre of the structure. In order to determine the new squares size, each base is rotated by 45 degrees and the appropriate rotation is made, so that the sides of the adjacent units are connected properly through one point. Once the two layers of the structure have been properly constructed, then by adding the diagonal cables and bars, the structure can be easily completed.

Ellipsoidal Tensegrity Structure

For the development of the ellipsoidal's second layer, a process similar to the one used for the sphere is followed. However, creating the second layer of the ellipsoid, involves a more complicated process, than the one used for the sphere, as in the ellipsoid the new offset surface has a different curvature and geometry than the first. This affects the connections between the square-bases. Unlike the case of the sphere, where the perpendicular axes of the units pass through the centre of the structure, in the case of the ellipsoid the axes do not pass through the center of the surface. Thus, new rules and requirements had to be determined and new techniques to be investigated, in order to create proper connections between the structural units.

Methods based on linear transformation processes, as well as a method that uses a nonlinear transformation procedure are applied and examined. These methods involve different ways of creating the offset surface that corresponds to the second layer of the structure. For example, the methods include two cases of shifting the initial layer's squares along their perpendicular axes that pass through the square's centroids: a) at a fixed distance and b) at a distance equal to the length of the side of each square have been examined (Figure 15). The offset distance represents the thickness of the structure, where in the first case the thickness is constant, while in the second case, the thickness of the structure changes proportionally. Lastly, a process using an optimization method has been applied, that estimates the vertical distance between the two bases of each unit, in order to achieve a more systematic solution. As in the case of the sphere, after the development of the two layers, the ellipsoidal structure can be easily completed by adding the diagonal cables and bars (Figure 16).

Based on the results, we highlight that the accuracy of all methods depends on the size of the greatest semi-axis of the ellipsoid, the thickness of the structure, as well as the number of tensegrity units that constitute the structure. We also notice that when some important

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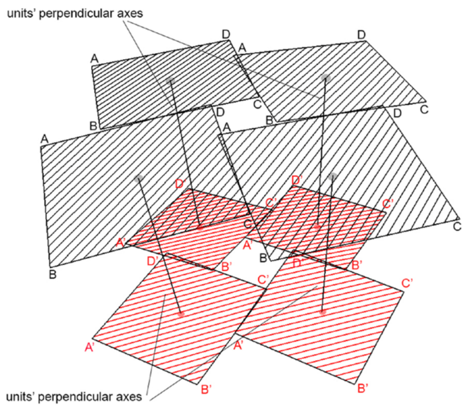

Figure 14. Square bases of adjacent units connected to each other

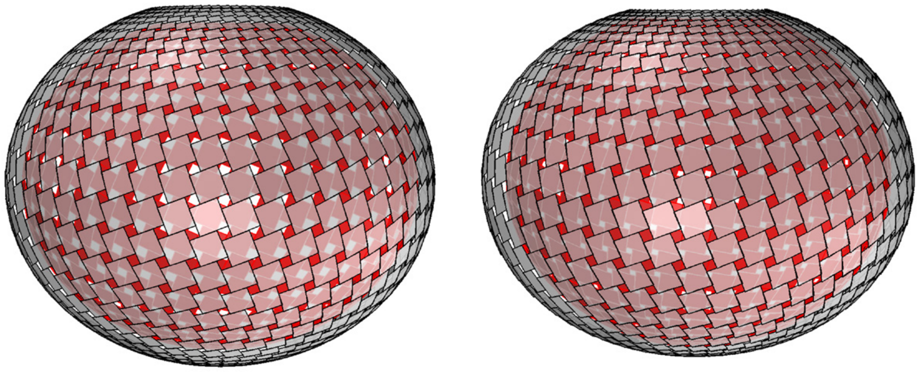

Figure 15. Creation of the second layer: a) at a fixed distance from the first layer and b) at a distance equal to the length of the side of each square

structural constrains and proportions are disregarded, the connections between the units are not made properly. Specifically, when the distance between the two layers is increasing more than the "allowed", narrow gaps are created between the units that are placed on the meridians. Proper procedures are developed, in order to control this factor and minimize these gaps when created. Concluding, more optimization methods can be applied in the future, for the further improvement of the developed techniques.

Applicability of the Method to the Catenoid and the Helicoid

The applicability of the developed method to certain types of minimal surfaces has also been examined. As in the case of the ellipsoid, two techniques are applied firstly; a) the first layer's squares have been shifted along their perpendicular axes at a constant distance, b) the squares have been moved to a distance equal to each square size. Both methods present interesting results. We notice that in the case of the catenoid, while we move from the central zone of the structure to the edges, a progressive growth of the unit sizes is occurred. Similarly, in the case of the helicoid, while we move from the helix that is nearest to the center to the helix boundary, the unit sizes are increased progressively (Figures 17, 18). Consequently, an optimization method has been applied defining the vertical distance between the bases of the units.

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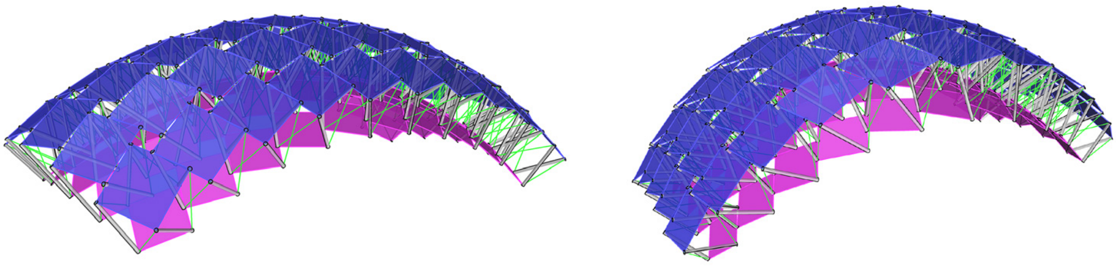

Figure 16.
Ellipsoidal configurations of the assembly of 5x10 square-base units. The in between cables are represented with green color and the bars with grey

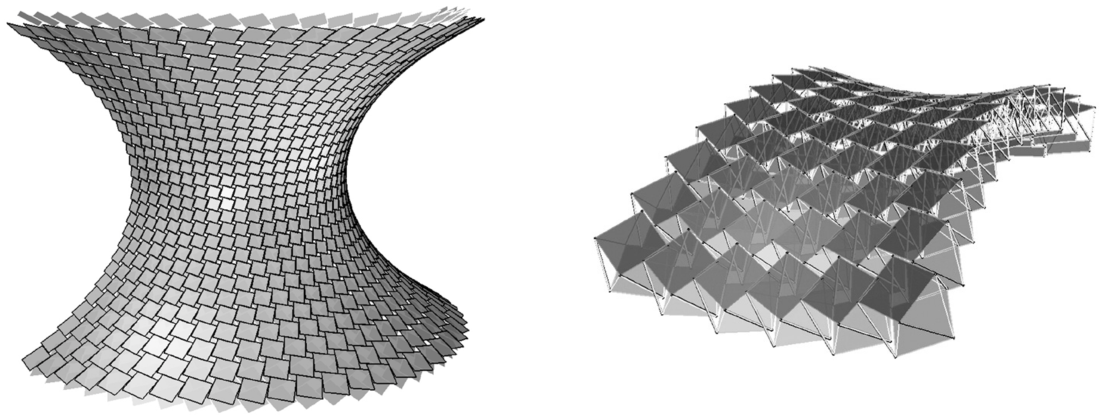

Figure 17.
Creation of the two layers and a 6x12 square-base units structure of a catenoid shape

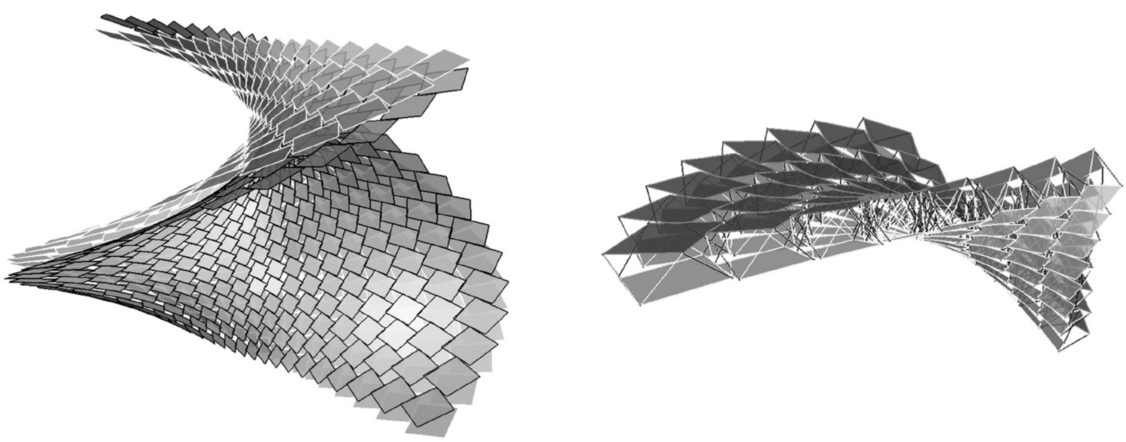

Figure 18.
Creation of the two layers and a 8x8 square-base units structure of a helicoid shape

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Conclusions

In this paper we have presented a method for the development of parametric models of double-layer tensegrity structures with double curvature consisting of square-base units. To this end, various methods have been investigated and have been applied to several curved surfaces including the ellipsoid and minimal surfaces. At the first stage of the research, we developed the first layer of the examined surfaces. A method that utilizes the Mercator projection was described and, in the case of the ellipsoid, geometric approximation processes were applied. We concluded that the accuracy of the method is not affected when a) the number of the squares on each row increases and b) the ratio between the two main axes of the ellipsoid changes. At a following stage, the second layer of the structure has been developed and various methods have been explored. Based on the results, we can conclude that the development of the second layer of the structure depends on the ratio of the unit size to geometric properties of the ellipsoid. The accuracy of the methods depends on the size of the greatest semi-axis of the ellipsoid, the thickness of the structure, as well as the number of the used tensegrity units. Optimization methods can also be applied, for further improvement of the various techniques. Our results demonstrate that the developed methods can be applied successfully to other curved surfaces such as the helicoid and the catenoid. Future research could focus on the investigation of ellipsoids, the dimensions of which change on more than one axes, as well as on the development of free-form surfaces parametric models. Furthermore, the study on form optimization methods comprises an interesting future research topic on environmental applications of such structures.

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